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On the Compatibility of Uncertainty Formalisms in Multi-Objective Optimization

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Abstract. Multi-objective optimization is a way to manage multiple objectives in analytical decision support systems. However, for real-life problems, different types of uncertainty often become prominent when defining the model. In this paper, we analyze these different types of uncertainties and suggest a suitable typology for a decision process based upon multi-objective optimization models. Uncertainty analysis can be performed based on the proposed typology; therefore, this analysis provides the necessary support for a decision maker in the identification the crucial uncertainty in the decision process.

Keywords. Decision Process, Multiple Objectives, Multi-Objective Optimization, Uncertainty

1. Introduction

When navigating in a world of large numbers of possible alternative solutions to a configuration or decision problem and each configuration/solution is to be assessed upon how it performs against different objectives, multi-objective optimization is a good candidate for the provision of configuring a system or providing interactive decision support. However, in many situations, the underlying system, which is to benefit from the optimization, is complex and its behavior is subject to uncertainty. Tackling this uncertainty and how to formally represent it is, however, a difficult but a crucial task in the development of optimization models aimed at improving the configuration of a complex system.

The concept of uncertainty and associated formalisms is reflected by many authors, and the various sources of uncertainty have not been left without discussion in the academic literature; see, e.g., (Knight, 1921), (Walker, et al., 2003), (Morgan & Henrion, 1992), (Kyläheiko, Sandström, & Virkkunen, 2002), (Matthies, 2007). Moreover, the authors have highlighted the importance of distinguishing between the different types and sources of uncertainty, and based upon this coping with them in different ways using different formalisms. From a decision process perspective, there is then a need for a comprehensive and integrated approach in order to manage uncertainty as part of the decision-making process. There are several studies devoted to this issue, for instance, (Walker, et al., 2003) synthesized uncertainty in model-based

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decision support whereas (Refsgaard, van der Sluijs, Højberg, & Vanrolleghem, 2007) described the uncertainty in the environmental modelling process.

Besides, many studies have addressed uncertainty types, as well as different models as a possible mapping between types and models in multi objective optimization domain, see (Fieldsend & Everson, 2005), (Aponte, et al., 2009), (Deb & Gupta, 2005). However, often the concentration is on specific areas, for instance evolutionary optimization, or on specific uncertainty types, such as epistemic and aleatory, or incorporating uncertainty into objective functions, or uncertain parameters. However, despite the recognition of various approaches towards the management of uncertainty in the multi-objective context, there is a lack of admissible methods that are able to distinguish between very different sources of uncertainty and to manage these different sources of uncertainty at a given time in a coherent manner. This motivates research on uncertainty models and multi-objective optimization from a holistic perspective. In this article, we analyze existing typologies of uncertainty and suggest the appropriate uncertainty typology in the decision process exploiting a multi objective optimization model.

This paper is organized as follows. Section 2 reviews various uncertainty typologies. Section 3 is devoted to the decision modeling process with multi-objective optimization, and a modeling process that is founded on multi-objective optimization is outlined. Section 4 presents an uncertainty typology suitable for the decision process that is executed based on multi objective optimization model. Section 5 draws conclusion and sketches the future work.

2. Related work on uncertainty

For many researchers, the concepts of uncertainty and risk are intertwined. Knight (Knight, 1921) described and emphasized conceptual distinction between certainty, risk and uncertainty. The concept of certainty refers to the complete knowledge, whereas the concept of uncertainty must be properly separated from the familiar concept of risk. The work by Knight serves as the way systematically to distinguish between risk and uncertainty where risk refers to randomness with knowable probabilities and uncertainty refers to randomness with unknown probabilities. As a consequence, the concept of uncertainty is reflected upon by many authors and the various sources of uncertainty have been subsequently discussed in the literature; (Walker, et al., 2003), (Morgan & Henrion, 1992), (Kyläheiko, et al., 2002), (Matthies, 2007).

In the light of systems approximated with complex mathematical models implemented in computer software, Kennedy & O'Hagan (Kennedy & O'Hagan, 2001) discussed prediction and uncertainty analysis and provide a classification. According to their classification, uncertainties in the computer based models can be divided into; 1) parameter uncertainty, 2) model inadequacy, 3) residual variability, 4) parametric variability, 5) observation error, and 6) code uncertainty. Based upon this classification, an approach using a Bayesian technique in order to fit the model to the observed data through adjusting the parameters is proposed. Taking into consideration these sources of uncertainty, the approach allows correcting some inadequacy between the model and the observed data through the best-fitting parameter values.

Relying on the initial source of Knight, authors (Kyläheiko, et al., 2002) treat the subject of uncertainties sources but from the dynamic capability context. The basic uncertainty categories suggested by these authors imply certainty, risk and uncertainty

as a starting point-of-departure for a classification. Following this, the sources of uncertainty are classified into two main categories, the *substantive* and *procedural*. Substantive uncertainty is related to possible outcomes and is depending on the environment while procedural uncertainty is related to the decision process and is depending on the decision makers. Within the substantive category, uncertainty can be further divided into parametric and structural. Parametric uncertainty refers to uncertain knowledge about parameters of the decision problem whereas structural uncertainty refers to imperfect knowledge about of structure of the decision problem. There are sources of uncertainty which are not being classified only to one of environment-related structural or chooser-related procedural uncertainties. It is needed the introduction of radical uncertainty concept, namely complexity, which includes both aforementioned types. The complexity concept is rooted in the "competence-difficulty" gap between the problem of choice and required competence introduced by Heiner (Heiner, 1983). In order to take into account structural uncertainty and complexity-related issues, (Kyläheiko, Sandström, & Virkkunen, 2002) developed a framework concerning a firm's governance structure choice.

Walker, et al. (Walker, et al., 2003) provide a conceptual foundation for the systematic management of uncertainty in model-based decision support activities, namely policy analysis, integrated assessment and risk assessment. According to the authors, there is neither a universally accepted terminology nor complete agreement on a typology of uncertainties, and as a possible solution to this, they proposed the defining the three dimensions of uncertainty, location, level and nature. In the paper, is described in detail each dimension and how these dimensions can be used to construct the uncertainty matrix. From the author's point of view, the location of uncertainty is identified by the logic of the model formulation, and the generic locations with respect to the model involve context, model uncertainty, inputs, parameter uncertainty and model outcome uncertainty. The uncertainty matrix's vertical axis identifies the location of uncertainty whereas its horizontal axis refers to two dimensions, the level and the nature. The matrix can be used for the classification of uncertainty at the different phases in decision support activities. Thus, this interdisciplinary theoretical tool allows systematically analysis of the uncertainty in model-based decision support.

Despite the existence of many sources of uncertainty, (Der Kiureghian & Ditlevsen, 2009) generally differentiate these sources into aleatory and epistemic. Epistemic uncertainty is characterized that it is possible to reduce it through a collection of new data or refining of the existing model. In contrast, aleatory uncertainty cannot be reduced. The authors discuss influences of these two types of uncertainty from the viewpoint of transparency in decision making. In a general view, this differentiation corresponds to the third dimensions of uncertainty, the nature, introduced by (Walker, et al., 2003) but in contradictory, the authors (Der Kiureghian & Ditlevsen, 2009) underline that the identification of nature of uncertainty allows for its reduction in developing sound risk and reliability models.

3. The decision modeling process

The widespread application of multi-objective optimization models is a challenging research topic since it is a way to involve and handle multiple and often conflicting objectives in a consistent fashion. The modeling process outlined in this section is rooted in the modeling processes of (Morgan & Henrion, 1992), (Belton &

Stewart, 2001), and (Grünert & Irnich, 2005) being adapted for multi-objective optimization modeling. An illustration of this process is presented in Fig. 1.

Similar to other decision process models, the modeling process in a multi-objective optimization context is started with identification of the real-world problem and defining the system boundaries of the system to be optimized. Since the system itself and the decision problems relative to improving the system are interconnected with several association and dependencies, one of the most important features of this modeling process is that it is an iterative process, cf. (Morgan & Henrion, 1992). Problem structuring comprises a set of different activities provided by different actors. These activities in a multi objective optimization context involve defining of goals, identification of objective functions and values as well as possible uncertainties associated with gathering data or objective functions. The principal aim is that, at the next phase, special attention should be given to the analysis of the various sources of uncertainty with respect to the system and the decision problems of concern. Then, gathering of data, data analysis and identification of possible uncertainty sources come in the synthesis. The phase of model building starts with building of a verbal model in order to give more detailed description of problem structure. The phase of multi objective optimization model involves the mathematical formulation of the verbal model. In this phase, knowledge, experience and elicited preferences can have a significant impact on the final decision. The algorithm's phase covers several aspects such as selection of optimization method, selection of preference modeling method, selection of solvers while the program's phase corresponds to execution of the built model with selected methodology. The greatest impact in the modelling process may come from the sensitivity and robustness analysis which provide valuable insights about obtained solution; moreover, the input value in a model can come from the sensitivity and robustness analysis. Thus, this modeling process allows iterative refinement during a decision process with using multi objective optimization.

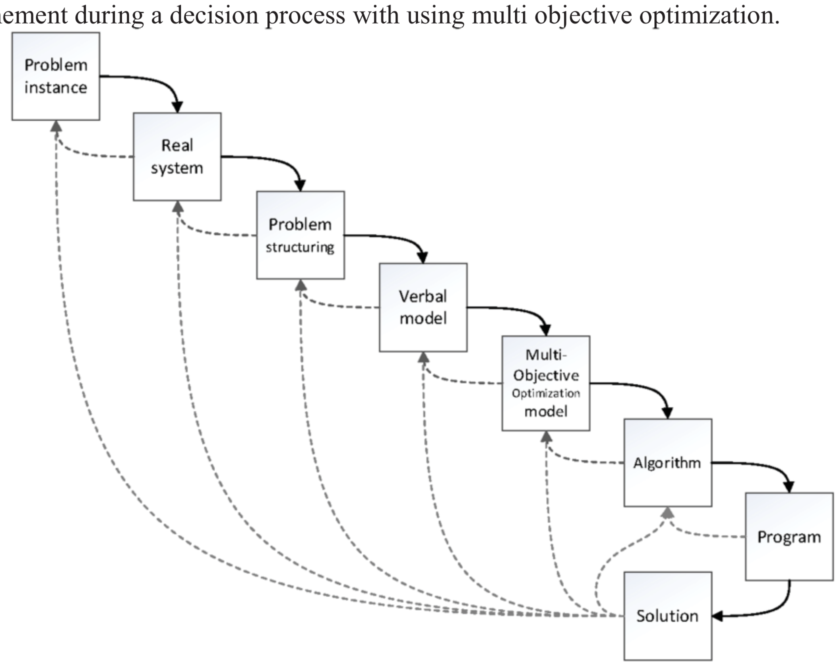

Figure 1 The decision process with multi objective optimization

4. Uncertainty typology in decision process with multi-objective optimization

The various classification schemes of uncertainty in section 2 describe some possible sources of uncertainty from different perspectives. In this section, relying on the above classification schemes, we summarize the different types of uncertainty which are associated with various phases of the modeling process outlined in section 3. There are two important reasons why it is not possible to directly map these classifications with a multi-objective optimization decision process. The first, uncertainty sources are considered independent from the modeling process, see, e.g., (Fieldsend & Everson, 2005), (Aponte, et al., 2009), (Deb & Gupta, 2005). The second, there is no grounded discussion with respect to how the uncertainty should be managed and formalized depending on their origins and in which phase of the modeling process this should be done.

In this section, we therefore provide a classification of the uncertainty types in the context of multi-objective optimization. Of importance is that the modeling process should involve uncertainty analyses based on conceptual differences between types of uncertainty measures. The modeling exercise is extended with the identification of possible sources of uncertainty and heavy emphasis is put on the iterative refinement during the modeling process. Careful examination of possible uncertainty sources is done in each phase of the decision modeling process. In order to mapping uncertainty with a decision modeling process exploiting multi objective optimization, the three dimensional concept of uncertainty of (Walker, et al., 2003) is a promising starting point. Following this conceptualization, we can encapsulate the *location*, *nature* and *level* of the uncertainties inherent. Table 1 provides a list of uncertainty locations and example sources of uncertainty in multi objective modeling process.

4.1. Sources (locations) of uncertainty in a multi objective optimization modeling process

The first step is to identify the sources (locations) of uncertainty in view of decision modeling process. After the identification of locations of uncertainty sources, the second level-dimension and third nature-dimension can be deliberated. We adapted and extended Walker methodology to the multi objective modeling process, and the summary of uncertainty locations with corresponding examples is presented in Table 1. The locations of uncertainty are categorized into problem context, model structure, model technical, input, parameters and model output.

4.1.1. The problem context

The problem context's identification refers to the phases from problem issue to a real system and from a real system to problem awareness. Context is formed from economic, environmental, political, social and technological prerequisites. Context defines the boundaries of the system with following consideration of awareness of the problem. The process to elicit subjective judgments and ideas about problem context from different actors of the modeling process is a way to construct a verbal model that should fit the behavior of the system. This process may include a group of stakeholders or decision-makers or experts, and thus the initial uncertainties are obtained from the elicitation of opinions of these participants.

4.1.2. Model structure

The next location is model uncertainty that refers to two types of location; model structure uncertainty refers to the phase from problem and to the verbal model while the model technical uncertainty can appear in the phase from algorithm to the program output. Model structure uncertainty in the multi objective optimization can involve the missing variable, the chosen model form or inaccurate form of the selected model. Modelling necessarily entails some simplifications, but over-simplification can render false conclusions about the solutions. For instance, omitting to include a variable can easily cause a model to show a positive relationship that does not hold in reality, thus falsely implying the impossibility of trade-offs.

The chosen model form can influence the outcome. In order to manage uncertainty, there are various models based on approaches such as stochastic programming, fuzzy mathematical programming, probabilistic programming, or a combination of these approaches, cf., e.g., (Azaron, et al., 2008), (Kalinina, et al., 2013). The inaccurate form of selected model refers to using of analytical or algorithmic expressions that are normally fitting a theoretical distribution to available data.

4.1.3. Model methodological

In a multi objective optimization context, a new category, model methodological can be added. This category involves two additional sub-categories that depend on the selected method. The first additional sub-category is defined by selection of multi-objective optimization method, the chosen method. The methods for solving multi-objective optimization problem can be divided into three categories, a posteriori approach, a priori approach and interactive approach (Deb, 2001, Reprinted June 2009), (Miettinen, 2001). In a posteriori approach, is assumed that the set of Pareto optimal solutions is first generated, and a decision maker should select the most preferred from this set. Therefore, the selection one the most preferred solution from the set of Pareto optimal solutions requires additional information about the decision maker's preferences. In a priori approach, is assumed that a decision maker articulates preference information and his/her aspirations in advance, and then the optimization process tries to find a Pareto optimal solution satisfying them. In interactive approach, it is assumed that an iterative solution algorithm is run and repeated, and, after every iteration, a decision maker is asked to specify preference information regarding to some given information. Knowledge about preference information from the decision makers is needed when using each algorithm. Accordingly, applying each of these approaches leads to a preference identification problem in the different phases of the decision process. Moreover, the different multi-objective optimization methods can give the different results. For instance, in case of using evolutionary multi-objective optimization algorithms as posteriori method, the obtained solution can be dominated and the real Pareto optimal set does not coincide with obtained (Miettinen, 2008).

The second additional subcategory is caused by selection preference modeling method. There are a lot of different methods to elicit preferences from a decision maker. The preference identification is needed before optimization when applying approaches are a priori or interactive while preference information is needed after optimization when applying approaches are a posteriori or interactive. In order to elicit weights, there are different ways depending on the chosen solution approach. A priori and interactive methods typically use multi criteria decision making techniques as a pre-processing tool. For this reason, there are the different ways to elicit weights, see

(Riabacke, et al., 2012). Whereas a posteriori and interactive methods can use the well-known techniques such as AHP (Saaty, 1980), PROMETHEE (Brans & Vincke, 1985), ELECTRE (Roy, 1991) as well as the specially developed method for elicitation preference involving ordering based on some pre-defined aggregation rule or use some visualization technique. The use of visualization techniques in case the decision maker is human are described by (Lotov & Miettinen, 2008), (Korhonen & Wallenius, 2008). The identification of some ordering of a large Pareto optimal set and based upon that ordering aid a decision making agent in its selection of one or some alternatives for further review are suggested in (Das, 1999a), (Kao & Jacobson, 2008), (Mattson, et al., 2004), (Kalinina, et al., 2013).

4.1.4. Input

According to (Walker, et al., 2003), the input location includes two sub-categories, namely external driving forces and system data uncertainty. The input uncertainty refers to the phase from multi objective optimization model to algorithm. The first subcategory, external driving forces, is defined by changes within the system that a decision maker is not be able to control. The objective function may be obtained through some numerical analysis or through physical experiment or extrapolation of a historical measurement. In this case, the objective function is modeled through the evaluated points in order to fit a function. The meta-modeling is used to help predict the value of future search point in order to decrease the number of evaluations. The multi objective version of the meta-modeling problem is devoted in (Knowles & Nakayama, 2008). However, the constructed objective function may differ from the genuine; this leads to uncertainty in model input. The second subcategory, system data uncertainty, is defined by lack of knowledge of the system's properties. So a lack of information about the initial conditions of the system is an example of input uncertainty.

4.1.5. Parameters

The parameters are various constants incoming in the model, and they can be divided into a priori chosen and calibrated parameters. The parameters uncertainties refer to the phases from the multi objective optimization model to algorithm when using a priori or interactive approaches and from program to the solution when using a posteriori or interactive approaches. If a priori chosen parameter is obtained from statistical analysis of observed data, then uncertainty of this parameter refers to the size of available samples of observations. The other example of parameter uncertainty are weighting coefficients, in this parameter is incorporated prior information from subjective human opinions. In addition, parameters uncertainty in the form of the weighting coefficients can be categorized as calibrated parameters in the case using of a posteriori or interactive methods. The nature of parameters uncertainties is epistemic normally; there is the possibility for reducing this type of uncertainty by future gathering of information (Der Kiureghian & Ditlevsen, 2009).

4.1.6. Model technical

The model technical covers technical aspects of model implementation and solving and it is depending on the computer and software. Different commercial or free software packages are often used in the multi objective optimization; however, the possible technical errors of used software package are generally not taken into account

during the decision modeling process. Moreover, the code uncertainty can occur in practice due the output of the code is unknown for given inputs. The model technical uncertainty is discussed by (Walker, et al., 2003) and (Kennedy & O'Hagan, 2001).

4.1.7. Model outcome

The next location uncertainty, model outcome, refers to the phase from program output to solution. All aforementioned locations influence model outcome uncertainties in the modeling process. Given that all antecedent locations are identified, we hereby determine the model outcome uncertainty. Moreover, some model outcome uncertainty can be caused by possible interaction between different types of uncertainties during the modeling process.

Table 1 Source of uncertainty in decision process with multi objective optimization

Sources of uncertainty	**Examples sources of uncertainty**
Problem context i. Technical and social	i. Knowledge, experience, attitudes of stakeholder(s) or decision maker(s) or expert(s)
Model structure i. the missing variable ii. selected model	i. missing some objective ii. stochastic or probabilistic or fuzzy model
Model methodological i. chosen multi objective optimization method ii. chosen preference modeling method	i. a posteriori, a priori and interactive approaches; ii. multi criteria decision making techniques or visualization techniques or special methods
Input i. external driving forces ii. system data uncertainty	i. meta-modeling, chosen distribution for parameters or objective functions ii. initial conditions
Parameters i. a priori chosen ii. calibrated	i. estimated parameters, weighting coefficients ii. weighting coefficients
Model technical computer dependencies	i. bugs in software, numerical approximation, solver
Model outcome	i. Outcome of possible combinations of uncertainty sources

4.2. Level of uncertainty

Inasmuch as preference elicitation from the decision maker plays a crucial role in the multi objective optimization; we follow level's classification suggested by (Refsgaard, et al., 2007). According to this classification, uncertainty can be categorized into four levels, statistical, scenario, qualitative and recognized ignorance. The first level, statistical uncertainty, can be characterized probability or number. In scenario level can be described by possible future development of the system or driving forces; however, future development is not well understood that led to difficult formulate it in the probability form. Qualitative uncertainty reflects the degree of confidence that a decision maker has about possible outcomes or probabilities of these

outcomes. Recognized ignorance level of uncertainty is characterized by lack of functional relationship, statistical property and scientific basis for developing scenario.

4.3. Nature of uncertainty

Following (Der Kiureghian & Ditlevsen, 2009) and (Aponte, et al., 2009), we define two subcategories of the third dimension, the nature of uncertainty, namely, epistemic and aleatory. Epistemic refers to imperfection of knowledge and can be reduced through additional research and empirical efforts while aleatory is defined as the inherent uncertainty or randomness which is dependent on external inputs of data, functions, parameters and model structures. Aleatory uncertainty can be described as a random value with known distribution. Taking into account multi objective context that requires involving linguistic expression, epistemic uncertainty covers epistemic linguistic uncertainty. For identification of the nature of uncertainty, we refer to (Der Kiureghian & Ditlevsen, 2009).

There are several theoretical methodologies in order to represent the nature of uncertainty. According to Dubois (Dubois, 2007), probability theory is in general used for representing two types of phenomena, the first is randomness which captures variability through repeated observations and the second is a belief which describes an individual's opinion on the occurrence of some event. In order to represent uncertainty, probability distributions and sets are used as the main tools. Probability distributions are suitable for expressing variability while sets are usable for representing incomplete information. Set-valued representations of partial knowledge can be in the form of interval or in the form of classical logic. Intervals can be used for representing incomplete numerical information whereas classical logic representation is good for representing incomplete symbolic information. Theories blending intervals and probability include imprecise probability theory (Walley, 1991), Dempster-Shafer theory (Ferson, et al., 2003), fuzzy sets (Klir & Folger, 1987). From a practical viewpoint, uncertainty representations involve fuzzy intervals, probability intervals, probability boxes, generalized p-boxes and clouds. In the Table 2, suitable methodologies are presented in order to deal with epistemic, epistemic linguistic and aleatory uncertainties.

Table 2 Suitable methodologies to representation of epistemic, epistemic linguistic and aleatory uncertainties

Nature of uncertainty	**Theoretic methodology**
Epistemic uncertainty	Classical Probability, possibility
Epistemic linguistic uncertainty	Fuzzy logic, imprecise probability
Aleatory uncertainty	Imprecise probability, possibility

5. Conclusion and further work

We analyze several uncertainty typologies described in literature and propose a typology of uncertainty in multi-objective optimization decision process. This typology follows the three dimensions concept of uncertainty of (Walker, et al., 2003). We suggest that in multi objective optimization context, an additional category of source of uncertainty, model methodological, is needed for the location dimension, source of uncertainty. Model methodological comprises two subcategories, chosen multi objective optimization method and chosen preference modeling method. We emphasize that the locations of uncertainty can be mapped with multi objective optimization

decision process, and, in the future, it allows integration of uncertainty management in the decision process in order to support decision makers during this process. In the level dimension, the categorization of suggested typology follows (Refsgaard, et al., 2007) four subcategories, namely, statistical, scenario, qualitative and recognized ignorance. The qualitative subcategory is needed due to the crucial role of subjective opinion of a decision maker in the optimization process. The nature of uncertainty is categorized as epistemic and aleatori, where epistemic linguistic is a part of epistemic uncertainty. Furthermore, we have briefly reviewed several theoretical frameworks for handling of uncertainty and mapped them with nature of uncertainty. Such a typology can aid in determining appropriate forms of representation uncertain entities and reduce possible uncertainties during a decision process.

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